A Revised Underwater Image Formation Model

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Abstract

The current underwater image formation model descends from atmospheric dehazing equations where attenuation is a weak function of wavelength. We recently showed that this model introduces significant errors and dependencies in the estimation of the direct transmission signal because underwater, light attenuates in a wavelength-dependent manner. Here, we show that the backscattered signal derived from the current model also suffers from dependencies that were previously unaccounted for. In doing so, we use oceanographic measurements to derive the physically valid space of backscatter, and further show that the wideband coefficients that govern backscatter are different than those that govern direct transmission, even though the current model treats them to be the same. We propose a revised equation for underwater image formation that takes these differences into account, and validate it through in situ experiments underwater. This revised model might explain frequent instabilities of current underwater color reconstruction models, and calls for the development of new methods.

1. Introduction

Researchers aiming to color correct underwater images are frequently faced with unstable results: available methods are either not robust, are too sensitive, or only work for short object ranges. This is usually explained by the challenge in the correction, e.g., images having low SNR, severe distortions and loss of color, etc. Here we suggest that there is a more fundamental reason to these instabilities than merely “low quality images”, and show that they stem from using an inaccurate image formation model.

Current underwater color correction methods [5, 8, 9, 12, 13, 26, 27, 31, 36, 40] rely on a commonly used image formation model first derived for haze [3, 4, 14, 17, 39]. However, light propagation in the ocean differs from that in the atmosphere in major ways, which renders this model inadequate when applied to underwater images.

In pure air, attenuation (sum of absorption and scattering) is only due to gas molecules and is an inverse function of the fourth power of wavelength [24, 32, 25] (Fig. 1a). Near the surface of the earth, however, air is mixed with solid and liquid particles (aerosols) that create the states we call clouds, dust, haze, smoke, smog, mist, fog and vog (fog from volcanic ash). In the presence of aerosols, whose mean diameters can be up to ten times larger than the incident light wavelength, multiple scattering effects become significant and wavelength dependency decreases [32]. In haze and fog, scattering becomes non-selective and attenuation
In the visible part of the electromagnetic spectrum becomes practically independent of wavelength [17, 29, 30, 32] (Fig. 1a).

In contrast to the atmosphere, natural bodies of water exhibit extreme differences in their optical properties (Fig. 1b). Some lakes are as clear as distilled water [15], some are pink [16], and others change color several times a year transitioning between white, blue, green, red, brown, and black [34]. In the ocean, coastal harbors are often brown and murky while offshore waters are blue and clear [1, 21]. Thus, a major difference between light propagation in the atmosphere and in natural waters is wavelength dependent attenuation.

Additionally, absorption in the atmosphere is generally negligible [22], but in the ocean its magnitude can be comparable to, and sometimes larger than scattering [28]. In fact, the wavelength dependency of attenuation in the ocean almost entirely comes from absorption (Fig. 1c). The ratio of scattering to attenuation coefficients affect the backscattered signal, which is the main cause of degradation in underwater images [35, 37]. Thus, whether the water body is absorption dominated or scattering dominated must be taken into account when modeling or estimating relevant color correction parameters.

These differences suggest that strong wavelength dependent attenuation may render the atmosphere-derived image formation model inadequate when applied to underwater scenes. Indeed we recently showed [1] that the direct signal obtained from the current underwater imaging model yields wideband attenuation coefficients (the projection of the physical attenuation coefficient onto the RGB domain) that depend not only on inherent properties of the water body, but also on the camera sensor, scene radiance, and imaging range. In most works, these dependencies are not taken into consideration. For example, in [5, 12, 27] wideband attenuation coefficients were estimated by picking values at single wavelengths from physical attenuation curves (like those in Fig. 1b), ignoring the sensor response of the camera used for imaging, and all the other dependencies we showed. In one case a sum on the discrete bands of the sensor response was used but only with one water type [26]. In [6, 7] the full wavelength-dependent model was used to model underwater photographs, but was not used for image correction.

Here, we expand our work in [1] and show that the current underwater image formation model actually introduces more dependencies that were ignored until now. Specifically, we show that the wideband coefficient for backscatter strongly depends on the veiling light (i.e., time of day and water depth), and whether the water body is absorption or scattering dominated. Most importantly, we show that the wideband attenuation coefficients for the direct signal and the backscattered signal are different, despite the fact that the current model assumes them to be the same. Through real-world experiments, we demonstrate the need for revising the image formation model. We perform an extensive error analysis comparing the current model to the revised one, and suggest how relevant coefficients can best be picked when all inputs to the revised model are not available. Our findings can explain the difficulties encountered thus far with image correction using the current model.

2. Light Transport in Scattering Media

2.1. Brief Oceanographic Background

In the ocean, inherent and apparent optical properties (IOPs and AOPs) govern light propagation (see Table 1 for variables and abbreviations used throughout the paper). The IOPs are only a function of the water constituents. These are beam absorption \(a(\lambda)\), beam scattering \(b(\lambda)\), and beam attenuation \(\beta(\lambda)\) coefficients, where \(a(\lambda)+b(\lambda)=\beta(\lambda)\). The AOPs depend on external factors such as the ambient light field [28], but are easier to measure than IOPs. Here the only AOP we use is \(K_d(\lambda)\), the diffuse attenuation coefficient of the downwelling light, which is the extinction experienced by light penetrating a water column vertically. The magnitude of \(K_d\) only weakly depends on ambient light, so it has been used to characterize the optical qualities of water bodies, such as Jerlov’s water types [21] (Fig. 1b,c).

Since both \(\beta\) and \(K_d\) are attenuation coefficients, which do we use in underwater computer vision? The answer is both, and their difference is important. While \(\beta\) governs the radiant power lost from a collimated beam of photons, \(K_d\) is defined in terms of the decrease of the ambient downwelling irradiance with depth, due to all photons heading in the downward direction in a diffuse (or, uncollimated) manner [28]. Generally, \(\beta\) is 2-5 times greater than \(K_d\) [23], and both contribute to the effective attenuation captured in a photograph, as we discuss in the next section.
When viewing direction is horizontal ($\theta = 90^\circ$; $d = d_0$), Eq. 1 becomes a function of only one attenuation coefficient ($\beta$) and simplifies to:

$$L(d_0; \xi; \lambda) = L_0(d_0; \xi; \lambda) e^{-\beta(\lambda) z} + L_*(d; \xi; \lambda) \frac{L_0(d_0; \xi; \lambda) \cos \theta}{\beta(\lambda) - K_d(\lambda) \cos \theta} \left(1 - e^{-\beta(\lambda) z} \right)$$

(2)

When $\theta \neq 90^\circ$, however, we are reminded of one of the reasons why color reconstruction of underwater images is challenging: depending on the viewing angle $\theta$, the magnitude of effective attenuation coefficient in a given scene changes! It can range from $[\beta(\lambda) - K_d(\lambda)]$ when looking up, improving visibility, to $[\beta(\lambda) + K_d(\lambda)]$ when looking down, significantly reducing it.

The first term in both Eqs. 1 & 2 is the **object radiance** resulting from photons traveling directly from the object to the observer, and the second term is the **path radiance**, which accounts for the photons reaching the observer from all directions. From here on, we will refer to them as $D$ for direct signal, and $B$ for backscattered signal. Then, Eq. 1 can be written as:

$$L = D + B.$$  

(3)

Note that in Eq. 1, we omitted the in-scattering term (also called forward scattering, $F$). This term would have represented the light that was reflected from the object away from the line of sight (LOS), but through re-scattering, got realigned at small angles along the LOS. Authors in [35] showed quantitatively that $F \ll D$, and it does not contribute significantly to the degradation of an image.

### 2.3. Backscattered Signal, B

We investigated the direct signal in Eq. 1 in [1]. Here, we focus on backscatter [35], also called path radiance [33].

Particles in the medium scatter the light incident on them in many other directions, acting as sources of light. Backscatter is the signal formed by these photons reaching the observer carrying no information regarding the scene that is being viewed. In Eq. 1, the path radiance $L_*$ is the radiance gained along a direction $\xi$ owing to scattering into that direction from photons traveling in all other directions $\xi'$ [28]. The probability of a photon traveling in a given direction after hitting a particle is determined by the **volume scattering function** (VSF). The VSF (a fundamental IOP from which all other scattering coefficients are derived), changes based on the type and concentration of particulate matter in the water body, is difficult to measure, and has only generally been quantified for a clear lake and a turbid coastal harbor [28]. The integral of the VSF across all directions yields the total scattering coefficient $b(\lambda)$, which is the main parameter governing backscatter [17, 30, 35, 41], and is readily available for Jerlov’s water types [38].

Consider again an infinitesimally small disk of thickness $dz$, that is not on the LOS (upper disk in Fig. 2). The ra-
distance $dL$ scattered from this disk in all directions is given by [17, 20, 28, 33]:

$$dL(z, \lambda) = b(\lambda)E(d, \lambda)dz,$$  \hspace{1cm} (4)

where $E(d, \lambda)$ is ambient light at depth $d$, which is also the radiance incident on the disk in this case. Along the LOS, at a distance $z$ away from the object, the received radiance based on Beer’s Law of exponential decay becomes [1, 28, 33]:

$$dB(z, \lambda) = dL(z, \lambda)e^{-\beta(\lambda)z}.$$ \hspace{1cm} (5)

Substituting Eq. 4 in Eq. 5 and integrating with respect to $z$ from $z_1 = 0$ to $z_2 = z$ gives us the backscattered signal as a function of wavelength $\lambda$:

$$B(z, \lambda) = \frac{b(\lambda)E(d, \lambda)}{\beta(\lambda)} \left(1 - e^{-\beta(\lambda)z}\right).$$ \hspace{1cm} (6)

When $z$ is selected to be large enough, we can obtain the value of backscatter at infinity, also termed veiling light. Thus as $z \rightarrow \infty$:

$$B^\infty(\lambda) = \frac{b(\lambda)E(d, \lambda)}{\beta(\lambda)}.$$ \hspace{1cm} (7)

Then, the total signal $T$ at the observer is:

$$T = E(d, \lambda)e^{-\beta(\lambda)z} + B^\infty(\lambda)(1 - e^{-\beta(\lambda)z}).$$ \hspace{1cm} (8)

### 2.4. Working in Camera Space

Now, we assume the observer has a camera with spectral response $S_c(\lambda)$ where $c = R, G, B$ represents color channels. The signal in Eq. 8 is integrated to obtain the intensity of the image formed at the sensor at a horizontal distance $z$ away from the object:

$$I_c = \frac{1}{\kappa} \int_{\lambda_1}^{\lambda_2} S_c(\lambda)\rho(\lambda)E(d, \lambda)e^{-\beta(\lambda)z}d\lambda + \frac{1}{\kappa} \int_{\lambda_1}^{\lambda_2} S_c(\lambda)B^\infty(\lambda)(1 - e^{-\beta(\lambda)z})d\lambda,$$ \hspace{1cm} (9)

where $\rho(\lambda)$ is the reflectance spectrum of the object, $\kappa$ is a scalar governing image exposure and camera pixel geometry [19], and $\lambda_1$ and $\lambda_2$ define the bounds of integration over the electromagnetic spectrum.

At depth $d$, the unattenuated image $I_c$ is:

$$I_c = \frac{1}{\kappa} \int_{\lambda_1}^{\lambda_2} S_c(\lambda)\rho(\lambda)E(d, \lambda)d\lambda.$$ \hspace{1cm} (10)

The veiling light $B^\infty_c$ as captured by the same sensor is:

$$B^\infty_c = \frac{1}{\kappa} \int_{\lambda_1}^{\lambda_2} S_c(\lambda)\frac{b_cE_c}{\beta_c}d\lambda.$$ \hspace{1cm} (11)

### 2.5. The Current Underwater Imaging Model

The current underwater imaging model for ambient illumination assumes camera response to be delta functions $(S_c(\lambda) = S_\delta(\lambda))$, or alternatively, attenuation to vary negligibly with wavelength. Accordingly, Eq. 9 is simplified to [4, 8, 12, 17, 26, 30, 35]:

$$I_c = J_c \cdot e^{-\beta(\lambda)z} + B^\infty_c \cdot (1 - e^{-\beta(\lambda)z}),$$ \hspace{1cm} (12)

similarly to the atmospheric dehazing equation. Here $\beta_c$ are the wideband ($R, G, B$) attenuation coefficients.

### 3. The Need for a Revised Model

#### 3.1. Dependencies of Attenuation Coefficients

We showed in [1] that the wideband attenuation coefficients $\beta_c$ estimated from the direct signal approximated by Eq. 12 have implicit dependencies on sensor response $S_c(\lambda)$, imaging range $z$, scene reflectance $\rho(\lambda)$, and irradiance $E(\lambda)$. From this point on, we label this coefficient $\beta^{D}_c$ to indicate that it has been derived from the direct transmission ($D$) term. From Eq. 12, it is given as [1]:

$$\beta^{D}_c = \ln \left[ \frac{D_c(z)}{D_c(z + \Delta z)} \right] \Delta z.$$ \hspace{1cm} (13)

Evaluating the first term in Eq. 10 with $z_1 = z$, and $z_2 = z + \Delta z$, we obtain [1]:

$$\beta^{D}_c = \ln \left[ \frac{\int_{\lambda_1}^{\lambda_2} S_c(\lambda)\rho(\lambda)E(\lambda)e^{-\beta(\lambda)z}d\lambda}{\int_{\lambda_1}^{\lambda_2} S_c(\lambda)\rho(\lambda)E(\lambda)e^{-\beta(\lambda)(z + \Delta z)}d\lambda} \right] \Delta z.$$ \hspace{1cm} (14)

Now we examine the backscattered signal. Following Eq. 9, the backscatter at the sensor at a distance $z$ is:

$$B_c = \frac{1}{\kappa} \int_{\lambda_1}^{\lambda_2} S_c(\lambda)B^\infty(\lambda)(1 - e^{-\beta(\lambda)z})d\lambda.$$ \hspace{1cm} (15)

We equate this exact equation to the backscatter term of the current underwater imaging model given by Eq. 12:

$$B_c(z) = B^\infty_c (1 - e^{-\beta_c z}).$$ \hspace{1cm} (16)

Thus, the wideband backscatter coefficient from Eq. 12 is:

$$\beta^{B}_c = \ln \left( 1 - \frac{B_c(z)}{B^\infty_c} \right) \Delta z.$$ \hspace{1cm} (17)

Substituting Eq. 15 into Eq. 17 yields

$$\beta^{B}_c = -\ln \left( 1 - \frac{\int_{\lambda_1}^{\lambda_2} S_c(\lambda)B^\infty(\lambda)(1 - e^{-\beta(\lambda)z})d\lambda}{\int_{\lambda_1}^{\lambda_2} B^\infty(\lambda)S_c(\lambda)d\lambda} \right) \Delta z.$$ \hspace{1cm} (18)
We can see that $\beta_c^D$ depends on the sensor response $S_c(\lambda)$, range $z$, and the veiling light $B_c^\infty$, which depends on the scattering and attenuation coefficients $b(\lambda)$ and $\beta(\lambda)$, and ambient light $E(\lambda)$ (Eq. 11). Comparing Eq. 17 with the direct signal effective attenuation

$$\beta_c^D = -\ln \left( \frac{I_c(z) - B_c(z)}{J_c} \right) / z ,$$

reveals that the effective wideband coefficients $\beta_c$ from the two terms of Eq. 12 are theoretically different although they are currently treated the same.

3.2. Physically Valid Space of $\beta_c^B$

Next, we derive the physically valid space of $\beta_c^B$ analogous to that of $\beta_c^D$ in [1]. We use the spectral response of a Nikon D90 and assume CIE D65 at the surface. We use Eq. 15 to calculate $B_c$, and Eq. 17 to extract $\beta_c^B$ for $z$ values ranging from 1m to the veiling light distance (i.e. when backscatter saturates). Values of $b(\lambda)$ were taken from [38]. Fig. 3a shows $B_c$ for each water type at 2m depth, where each ‘x’ denotes the veiling light $B_c^\infty$ calculated using Eq. 11. Note that the distance at which veiling light is reached can range from 10s to 100s of meters depending on the attenuation coefficient of the water (Fig. 3b).

Fig. 4a shows that $\beta_c^B$ changes very little with $z$ for a given water type (filled circles). This allows us to use the mean $\beta_c^B$ value as a representation of each water type. We fit two lines to these $\beta_c^B$ means in 3-space: one for clear water where attenuation is dominated by absorption (Fig. 1c; I-IB), and one for water types where scattering is more dominant (Fig. 1c; II, III, 1-9C). These lines denote the locus of $\beta_c^B$ in 3-space (Fig. 4b).

The magnitude of veiling light $B_c^\infty$ is directly proportional to the ambient light (i.e., depth), which in turn, causes the locus of $\beta_c^B$ to move (Fig. 4c,d). Finally, we note that while the locus of $\beta_c^D$ was shown to depend on camera sensor response, that of $\beta_c^B$ is less sensitive to it. This is likely because the backscattered signal is formed independent of the reflectance of objects in a scene, and depends, most strongly, on ambient light, which attenuates rapidly with depth. Fig. 4e shows the $\beta_c^B$ locus for 74 cameras (Arriflex, Canon, Casio, Fuji, Hasselblad, Kodak, Leica, Manta, Nikon, Nokia, Olympus, Panasonic, Pentax, Phase One, Point Gray, Sony, Sigma) for absorption-dominated water types; camera details and loci for scattering-dominated water types are given in Supplementary Material.

3.3. $\beta_c^D$ Vs. $\beta_c^B$

Fig. 4f&g compare $\beta_c^D$ and $\beta_c^B$ for the same scene in the same water. We simulated a diver at 5m, photographing two Macbeth charts placed horizontally at 1&10m from him. We calculated the RGB values of these charts using Eq. 9 with the response of a Nikon D90, assuming CIE D65 at the surface for clear oceanic water (Jerlov I, Fig. 4f), and a murky coastal harbor (Jerlov III, Fig. 4g). Then, using Eqs. 17&19, we extracted the wideband attenuation coefficients. As we showed in [1], $\beta_c^D$ also depends on the distance between the camera and the charts, so the charts cluster in different parts of the $\beta_c$ space. In contrast, there is one $\beta_c^B$ value per scene (black x’s), and it varies very little by distance. Yet, the value of $\beta_c^B$ for both water types is different than the $\beta_c^D$ values for both charts.

4. A Revised Image Formation Model

In light of our findings in Secs. 3.1 & 3.2, we propose the following revised underwater image formation model:

$$I_c = J_c e^{-\beta_c^D(v_D)z} + B_c^\infty \left( 1 - e^{-\beta_c^B(v_B)z} \right).$$

Here, the vectors $v_D$ and $v_B$ represent the coefficient dependencies $v_D = \{ z, \rho, E, S_c, \beta \}$ and $v_B = \{ E, S_c, b, \beta \}$. To parallel Eq. 12, we also only considered the horizontal imaging case, but extension to other directions as given in Eq. 1 is straightforward.

5. Validation Via Real-world Experiments

5.1. Backscatter Estimation From Photographs

The image signal $I_c$ in Eq. 9 is exactly equal to backscatter $B_c$ for a perfect black object ($\rho = 0$). While efforts have been made to manufacture a surface close to a perfect black [18], these materials are not commercially available in small quantities. Thus, we used a color chart and leveraged the fact that the gray patches reflect light uniformly at different percentages in a linear image [2]. We fit a line to the RGB intensities corresponding to each patch using their known luminances (chromaticity coordinate $Y$). The value of this line at $Y = 0$, which would have been the luminance of perfect black (Fig. 5a), is the $B_c$ for that color channel.
We conducted underwater experiments in the Red Sea and the Mediterranean (Fig. 6). For each experiment, we laid out 5 colors charts at horizontal distances of 1,3,5,7 and 9m from the camera. In the Red Sea, we tested a Nikon D810 and a Sony RX100 simultaneously (Fig. 6a), taking 5-10 photographs of scenes at 2&6m depth. In the Mediterranean, we photographed scenes at 6&10m only with a Nikon D810. All images were taken in raw format, in manual mode, keeping the settings consistent for each camera for a given scene.

From each photo, we calculated $B_c$ as described in Sec. 5.1, and extracted $\beta_c^B$ using Eq. 17. We did not have the Nikon D810 response, so used the mean response of all Nikons from Fig. 4e to draw the loci in Fig. 6b, and used CIE D65 for surface light. The loci for the Red Sea 2&6m scenes (Fig. 6b) were different as predicted. The $\beta_c^D$ extracted from both Nikon (green dots) and Sony photos (red dots) aligned closely with the ‘mean Nikon’ locus; each dot represents the average coefficient ($\beta_c^B = [0.32, 0.22, 0.18]$ for 2m, and $\beta_c^D = [0.29, 0.25, 0.2]$) of five color charts in each photo.

The locus of $\beta_c^D$ is expected to move based on sensor [1]. We did not have Sony RX100 response, so in Fig. 6c we show $\beta_c^D$ locus for ‘mean Nikon’, for water type II, for distances varying from 1-9m (matching our experiments). Each filled circle represents $\beta_c^D$ for a given color patch at a given distance in the scene. As with $\beta_c^B$, experimental data validate the expected locus of $\beta_c^D$, and as predicted, $\beta_c^B$ and $\beta_c^D$ do not have the same values.
6. Error Analysis: Current Vs. Revised Model

While the model in Eq. 20 is the most accurate, we realize it is difficult to obtain all its parameters given their dependencies. Therefore, we performed an extensive error analysis to quantify the effect of every part in the revised model. We tested seven scenarios of color reconstruction using the current (Eq. 12) and revised (Eq. 20) models with different parameters:

S1. Current model with $\hat{\beta}_c = \beta^B_c$.
S2. Current model with $\hat{\beta}_c$ derived from the chart at $z = 1$ m (i.e., $\hat{\beta}_c = \beta^D_c (z = 1$ m$))$.
S3. Current model with $\hat{\beta}_c = \beta^D_c (z = 5$ m$)$.
S4. Revised model with the correct $\beta^B_c$ and $\beta^D_c (z = 1$ m$)$.
S5. Revised model with the correct $\beta^B_c$ and $\beta^D_c (z = 5$ m$)$.
S6. Revised model with the correct $\beta^B_c$ and $\beta^D_c (z)$, where $\beta^D_c (z)$ is obtained from the average of $\beta^D_c (z, \rho)$ values for all color patches for each chart at a given $z$.
S7. Revised model with the correct $\beta^B_c$ and $\beta^D_c (z, \rho)$, where $\beta^D_c (z, \rho)$ is the coefficient calculated for each color patch at each range.

We synthesized a scene using Eq. 9 where a diver at 2 m depth is photographing 5 color charts horizontally in front of him at $z = 1, 3, 5, 7, 9$ m. For all simulations we used $b$ and $\beta$ of Jerlov II, $S_r$ of a Nikon D90, took $E(0, \lambda)$ to be CIE D65, and the reflectances $\rho$ of a Macbeth chart. The ground truth $\beta_c$ corresponding to this simulation are shown in Fig. 7a. Error between the unattenuated color $J$ (Eq. 10) and the reconstructed one $\hat{J}$ (found either inverting Eq. 12 or 20) is measured as the dissimilarity $\alpha$ between them in RGB space (Fig. 7b):

$$\cos \alpha = J \cdot \hat{J} / (|J| \cdot |\hat{J}|).$$

6.1. Corrections Using The Current Model

For all patches, errors resulting from using the current image formation model using only $\beta^D_c$ are consistent across range $z$ (S1, black curves in Fig. 7b), except for colored patches where they slightly increase with $z$. In contrast, using the current model with $\beta^D_c$ extracted from only one of the charts at $z = 1$ m, averaged for all colors, yields errors that are small for $z = 1$ m but significantly increase with $z$ (S2). This is because the magnitude of $\beta^D_c (z = 1$ m$)$ is larger than $\beta^D_c$ at other $z$, and it also yields an incorrect backscatter calculation. However when the same correction is carried out using only $\beta^D_c (z = 5$ m$)$ (S3), the overall errors are reduced. This is because $\beta^D_c (z = 5$ m$)$ for every color channel is much closer to the mean $\beta^D_c$ in the scene, and also closer in magnitude to $\beta^B_c$, resulting in an acceptable backscatter removal. Thus, when using the current model errors are reduced when calibration of coefficients takes places not at proximity to the camera. Thus, if one is only able to estimate a single $\beta^D_c$ value, choosing coefficients from a mean $z$ might prevent extremely high or low values in the corrected image. Note that this suggests that estimating coefficients from color charts that are close to the camera, as is often done, is a bad habit. Yet, S3 still yields visible errors in the hues of the reconstructed colors, especially for large $z$.

6.2. Corrections Using The Revised Model

If backscatter is removed using the correct $\beta^B_c$, but the colors are reconstructed using the $\beta^D_c$ extracted only from the first chart (S4), errors remain almost identical to S2, implying that $\beta^D_c$ has a more prominent role in color reconstruction than $\beta^B_c$. The only noticeable improvement is in the reconstruction of black, but the errors still increase with
z, likely due to the overcompensation from $\beta^D_c$ calculated at $z = 1m$. The overcompensation due to $\beta^D_c$ calculated at distances close to the camera is relevant for all water types (see Supplementary Material).

In S5, we extract $\beta^D_c$ from the middle chart placed at $z = 5m$ (yellow curves in Fig. 7b). This causes a drop in error when compared to S4 because as Fig. 7a shows, $\beta^D_c(z = 1m)$ greatly overcompensates the colors in the rest of the scene, whereas $\beta^D_c(z = 5m)$ is close to the average $\beta^D_c$ in the entire scene.

When $\beta^D_c$ is expressed as a function of $z$, the error magnitudes decrease significantly (S6). In this case, we calculated one $\beta^D_c$ value calculated per chart (i.e., at the $z$ corresponding to each chart), which is averaged over all reflectances $\rho$. For this scenario, the highest errors arise for the colors that contain red. This is because for those colors when $z > 5m$ the magnitude of $\beta^D_c(z)$ becomes less than that of $\beta^D_c(z = 5m)$, and $\beta^D_c(z = 5m)$ estimates the corresponding coefficient better. In contrast, for colors containing blue, when $z > 5m$, the opposite happens: $\beta^D_c(z)$ value becomes closer to the corresponding coefficients and the errors decrease when compared to S5. Yet, even with the higher errors associated with colors containing red, errors resulting from S6 are almost as low as errors resulting from S7 - a nearly perfect reconstruction using $\beta^D_c(z, \rho)$ (magenta curves in Fig. 7b).

In real life, having $\beta^D_c$ for every $z$ and $\rho$ for a scene is difficult, but can be done, for example, using structure-from-motion to group patches corresponding to the same objects at different known distances [8]. Then our loci calculation and simulation might be used to estimate the $\beta^D_c$ value per distance per object. Even if this scheme is not possible, our insights demonstrate the importance of choosing the best distance to estimate $\beta^D_c$ from.

### 7. Discussion

We demonstrated through theoretical analysis and real-world experiments that the commonly used underwater image formation model yields errors that were not accounted for thus far. We showed that the coefficient associated with backscatter varies with sensor, ambient illumination, and water type; and most importantly, it is different than the coefficient associated with the direct signal. These, together with dependencies we showed in [1], might explain many inaccuracies and instabilities in current algorithms. Our revised model will lead to the development of methods that will better correct complex underwater scenes.

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### References


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**Figure 7.** a) Ground truth values for $\beta^B_c$ (dashed line) and $\beta^D_c$ (lines colored according to the 24 patches). The value of $\beta^B_c$ almost does not change with $z$, while $\beta^D_c$ decreases logarithmically. b) Color reconstruction error for the seven cases listed in Sec. 6 using Eq. 21, and c) RGB visualization of errors. Each chart is white balanced using the white patch of the ground truth image.